Scientific Papers of
the Department of Electrical Power Engineering of
the Wrocław University of Technology

PRESENT PROBLEMS OF POWER SYSTEM CONTROL

7

Wrocław 2016

Cover design
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Printed in the camera ready form

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ISSN 2084-2201

Print and binding: beta-druk, www.betadruk.pl

signal processing, digital filter,
magnitude estimator, high speed overcurrent relay

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OPTIMIZATION OF CURRENT MAGNITUDE ESTIMATORS BASED ON MARQUARDT–LEVENBERG ALGORITHM

Digital filtering, correlation methods, time delay methods, signal differentiation are the most commonly used methods of estimating fundamental frequency orthogonal components utilized in magnitude estimators. The foundation for designing filters used in aforementioned methods are usually demanded frequency responses or signal models with their parameters to be estimated. A weak point of both approaches is frequency-domain modelling ignoring time-domain performance of the magnitude estimators. In order to fulfil the requirements of protection with the optimum speed for many different configurations, operating conditions and construction features of power systems, it is necessary to develop magnitude estimator design methods aimed at modelling with respect to high-speed response with simultaneous acceptable estimation accuracy in the steady state.

The article discusses the implementation of Levenberg–Marquardt algorithm to optimization of current magnitude estimators designed for power system protection with the focus on estimators used in instantaneous overcurrent relays. The paper presents details of optimizing algorithm, power system model used for acquisition of signal patterns and estimator performance analyzes.

1. INTRODUCTION

Current is the earliest protection quantity used in power system relaying. Modern numeric relays, equipped with digital filtering algorithms eliminating undesirable harmonics from protection signals, provide an accurate magnitude measurement of current and voltage fundamental components. However, the accuracy is sacrificed by the long duration of estimation transients after the disturbance inception.

The speed of current magnitude estimation is essential when concerned with instantaneous overcurrent relays referred to as High Speed (HS) overcurrent relays.

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These protections are used in medium voltage networks as main protections for clearing high level current faults and in some single-end fed 110 kV power lines.

HS protection calls first of all for short time magnitude estimation transients while ensuring adequate accuracy for the assumed signal model with its disturbances such as e.g. decaying DC component and harmonics occurring in the pre- and fault states.

The article presents the method of designing orthogonal filters optimized with respect to predefined magnitude estimator algorithm they are to be used in. The optimization process is determined by the adopted objective function, the training signal model resulting from faults modelled in the single-end fed 110 kV power line, and the model of harmonic distortions occurring in these networks in Polish power system.

Digital filter design is a multistep process in which one can specify the steps essential for the project and the final properties of obtained filters. The primary step is to determine the desired properties of the filter, which usually translates to determine the desirable complex frequency response. One should then make choose of filter class and determine the criterion according to which the choice of filter coefficients will be made. The last step is the choice of the best solution method for formulated task and its solution.

The desired magnitude or phase diagrams, or both these two diagrams, are the foundation of the project in most methods of FIR filter design. Sometimes, instead of the phase diagram the desired diagram of the filter group delay is proposed.

Usually, the basis for FIR filter design is a polynomial approximation. Filter design methods can be classified with respect to the complexity as computer aided filter design methods and traditional methods. The term *traditional methods* should be understood as methods in which the filter design is carried out without the use of iterative techniques, i.e. window methods based on inverse discrete Fourier transform [1]. The traditional methods are not computationally very complex, and their applications are limited to quite specific cases with respect to approximated characteristics. When using the traditional design, additional requirements relating to the properties of the filter in time or frequency domain cannot be taken into account. Therefore, this article presents the optimal estimator design method dedicated for measuring of the fundamental frequency voltage or current component magnitude.

2. MAGNITUDE ESTIMATORS

2.1. FUNDAMENTALS

One group of magnitude estimation methods is based on orthogonal fundamental frequency components of the analyzed periodic signals. According to the Pythagoras's theorem magnitude of sine wave can be calculated from the following formula

$$I(n)=\sqrt{i_c^2(n)+i_s^2(n)} \tag{1}$$

where i_c, i_s are orthogonal components evaluated for instant n. One can obtain orthogonal components, for instance of current i, by digital filtering with coefficients $b_{c(j)}$, $b_{s(j)}$ of the pair filters according to the following formulae

$$\begin{aligned} i_c(n) &= \sum_{j=0}^{N-1} i(n-j)b_{c(j)} \\ i_s(n) &= \sum_{j=0}^{N-1} i(n-j)b_{s(j)} \end{aligned} \tag{2}$$

These filters should provide a phase shift of π/2 and the same modules of output signals for the fundamental frequency. Moreover, the proper shape of the frequency response above and below the nominal power system frequency should be ensured. Usually impact of harmonics and decaying DC offset on orthogonal components estimation should be minimized. This very well define requirements of filtering protection signals as far as magnitude estimation accuracy in the steady state is concerned, however no specification are given explicitly regarding the interaction between magnitude and phase responses and the estimation transient state desired. The most commonly used filters that meet the basic requirements listed above are orthogonal sine-cosine filters referred to as full cycle Fourier filter. These filters introduce the estimation transient state of the length that is equal to the length of filter data window. Moreover, the impact of decaying DC current components on magnitude estimation based on these filters is significant.

2.2. ESTIMATOR DESIGN BASED ON TARGET PATTERN PRESENTATION

In the proposed method, two filters orthogonal only near the system frequency, are design by minimizing of the objective function that is expressed by the following formula

$$Q(\mathbf{B}_c,\mathbf{B}_s)=\sum_{k=1}^{L}\sum_{n=1}^{K}\frac{1}{4}W\big(M,e(\mathbf{B},k,n)\big)\big(I^2(\mathbf{B},k,n)-\hat{I}^2(k,n)\big)^2, \tag{3}$$

where

$$\mathbf{B}=[\mathbf{B}_c \quad \mathbf{B}_s]=[b_{c(0)}\; b_{c(1)}\ldots b_{c(N-1)} \quad b_{s(0)}\; b_{s(1)}\ldots b_{s(N-1)}]=[b_{(0)} \quad \ldots \quad b_{(2N-1)}] \tag{4}$$

is a parameter vector of the estimator.

For a given set of training cases, function Q takes value of the cumulative weighted squares of estimation errors. The error is defined as difference between the magnitude estimate squared and target estimate squared. An estimate error for n-th sample in k-th fault case is expressed as follows

$$e(\mathbf{B}, k, n) = I^2(\mathbf{B}, k, \mathbf{n}) - \hat{I}^2(k, n) \tag{5}$$

The square of (1) is used in (5) to avoid the computation of the partial derivative of the square root function.

The value of the objective function for a fixed set of training examples depends only on filter coefficients employed in the estimator (1-2). Sets $\mathbf{B}_c$ and $\mathbf{B}_s$ of 20 coefficients each constitute set $\mathbf{B}$ of parameters to be optimized.

The weighting function W in (3) defines the estimate transient state length (M) and steady state for $n > (M + N)$. This function eliminates from training process these data windows that are associated with sampling instants before the fault inception designated with N.

$$W(M, e(\mathbf{B}, k, n)) = \begin{cases} 1 & \text{for} \quad n \geq (M+N) \\ 1 & \text{for} \quad N+1 \leq n < M+N \quad \text{AND} \quad e(\mathbf{B}, k, n) > 0 \\ 0 & \text{for} \quad n < N+1 \end{cases} \tag{6}$$

Values taken by weighting function are shown in Fig. 1. The length of the transient state denoted by M is a parameter of the proposed estimator. The corresponding estimate is arbitrary for this state, but should not be greater than the magnitude at the steady state. This results in that only those moments of the transient state are used in the optimization, for which the error exceeds 1, i.e. when the overshoot of the magnitude estimate occurs.

In the proposed approach, current samples for all fault cases are normalized to provide the steady state magnitude equal to 1 as depicted in Fig. 1.

One group of iterative optimization techniques are methods based on the gradient of the objective function calculated with respect to the optimized quantities. In the proposed approach the gradient of (3) relative to the coefficients of $\mathbf{B}_c$ consists of components computed with the following formula

$$\frac{\partial Q}{\partial b_{c(l)}} = \sum_{k=1}^{L} \sum_{n=1}^{K} W(M, e(\mathbf{B}, k, n)) e(\mathbf{B}, k, n) i_c(\mathbf{B}_c, k, n) \frac{\partial i_c(\mathbf{B}_c, k, n)}{\partial b_l}$$
$$= \sum_{k=1}^{L} \sum_{n=1}^{K} W(M, e(\mathbf{B}, k, n)) e(\mathbf{B}, k, n) i_c(\mathbf{B}_c, k, n) i(k, n-l+1) \tag{7}$$

and components obtained with respect to coefficients in $\mathbf{B}_s$ with formula as follows:

$$\frac{\partial Q}{\partial b_{s(l)}} = \sum_{k=1}^{L}\sum_{n=1}^{K} W\left(M, e(\mathbf{B}, k, n)\right) e(\mathbf{B}, k, n) i_s(\mathbf{B}_s, k, n) i(k, n-l+1) \tag{8}$$

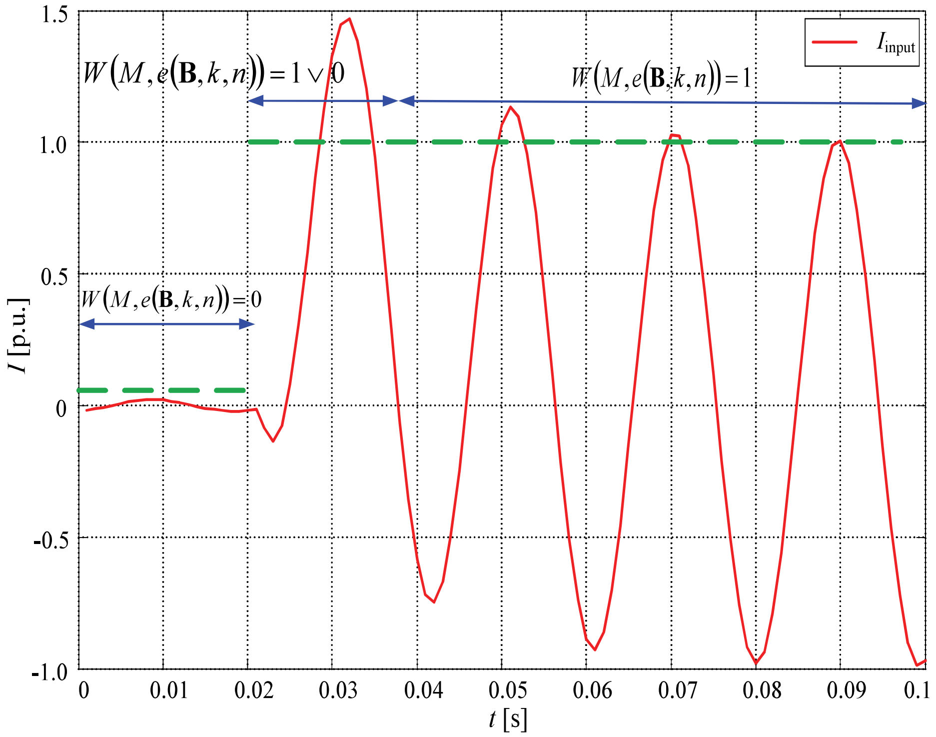

Fig. 1. Values taken by weighting function in pre fault, transient, and fault state

Since Levenberg–Marquardt method is adopted in this approach the approximation of Hessian of the following form

$$\mathbf{H}(\mathrm{Q}) = \begin{bmatrix} \frac{\partial^2 Q}{\partial b_{(0)}\partial b_{(0)}} & \cdots & \frac{\partial^2 Q}{\partial b_{(0)}\partial b_{(2N-1)}} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 Q}{\partial b_{(2N-1)}\partial b_{(0)}} & \cdots & \frac{\partial^2 Q}{\partial b_{(2N-1)}\partial b_{(2N-1)}} \end{bmatrix} \tag{9}$$

has to be determined. The components of Hessian are expressed by the following formula:

$$\frac{\partial^2 Q}{\partial b_{s(m)}\partial b_{c(l)}} = \sum_{k=1}^{L}\sum_{n=1}^{K} W\left(M, e(\mathbf{B}, k, n)\right) i_c(\mathbf{B}_c, k, n) i_s(\mathbf{B}_s, k, n) i(k, n-l+1) i(k, n-m+1) \tag{10}$$

Updating of the coefficients in row matrix B for *p*-th optimizing epoch is proceeded according to the rule proposed by Levenberg and Marquardt

$$\mathbf{B}(p+1)=\mathbf{B}(p)-\frac{\underset{\mathbf{B}(p)}{\operatorname{grad}Q}}{(\mathbf{H}(\mathrm{Q})+\lambda \mathrm{I})} \tag{11}$$

where I is the unit matrix and λ time-varying factor weighing between the Newton–Rapson method and the steepest descent method [2].

2.3. POWER SYSTEM MODEL

Training data set consists of examples of current waveforms and the corresponding target magnitudes taken from the steady state. Aforementioned waveforms were recorded for faults in 110 kV line modelled in EMTP (Fig. 2) under analog filtering with 330 Hz cutoff frequency and sampling frequency of 1 kHz. Additionally, training current waveforms were artificially distorted with harmonics of maximum magnitudes as given in Table 1.

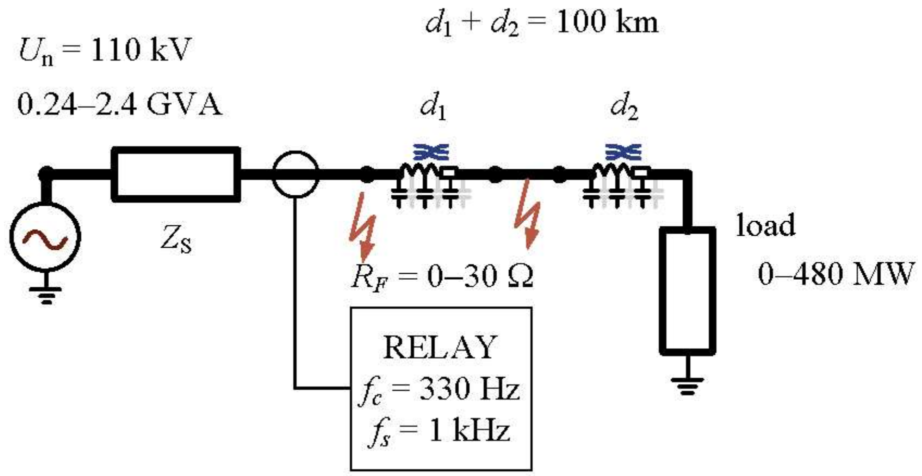

Fig. 2. Modelled 110 kV system

Table 1. Accepted levels of harmonic distortions for nominal power system conditions

$h_{\#}$	I [A]
2	40
3	70
4	25
5	30

3. RESEARCH OUTCOMES

The estimator (1-2) underwent optimization for different lengths of the transient states with parameter M varied from 0 to 20. The maximum relative errors of measurements with respect to maximum error of algorithm (1) based on the full cycle Fourier filter were evaluated for all received optimal estimators. The errors were compared on the basis of the same set of fault cases, yet unused for the optimization.

Figure 3 illustrates the unit impulse response of two filters obtained for the adopted 7 samples length of the transient state. As one can see, the windows of these filters are neither symmetric nor anti-symmetric, so that the phase characteristics of these filters are non-linear. However, the fundamental component magnitude estimation according to (1) does not require linearity of phase displacement in wide range of frequency yet only calls for both filters to have the same value of magnitude and constant relative phase shift equal to π/2 for close proximity of 50 Hz (47 Hz–52 Hz). Furthermore, both characteristics (phase and magnitude) for higher frequencies should be shaped to provide a compromise between the expected response rate estimator (1) and its accuracy (eliminating the impact of harmonics) for samples of steady state i.e. for n exceeding $M + N$. The frequency responses of both optimized filters for $M = 7$ are shown in Fig. 4.

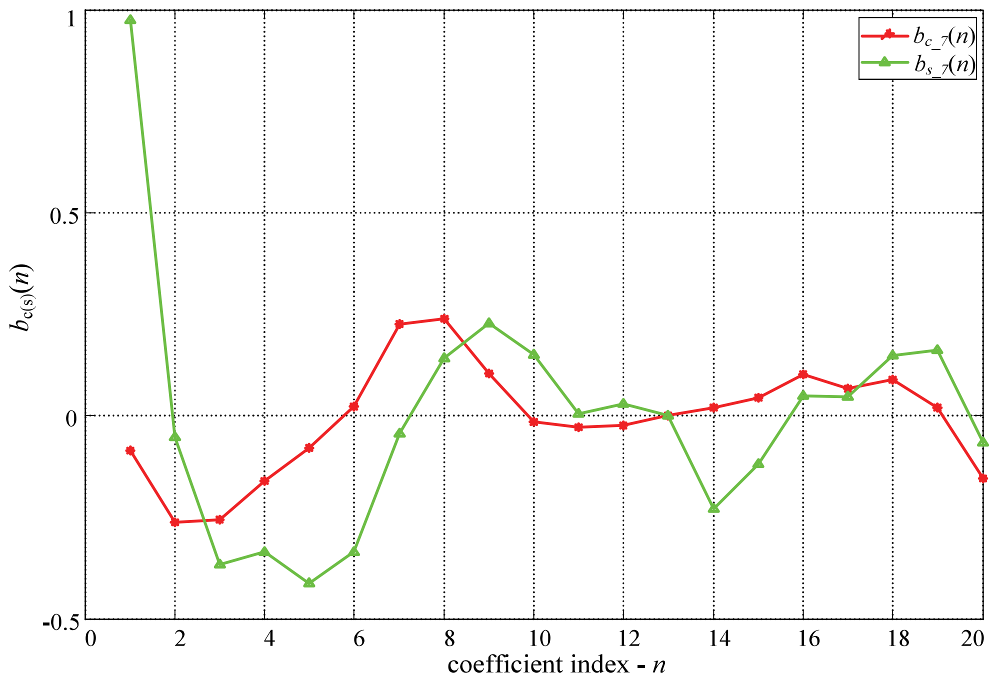

Fig. 3. Unit imulse response of filters optimized for $M = 7$

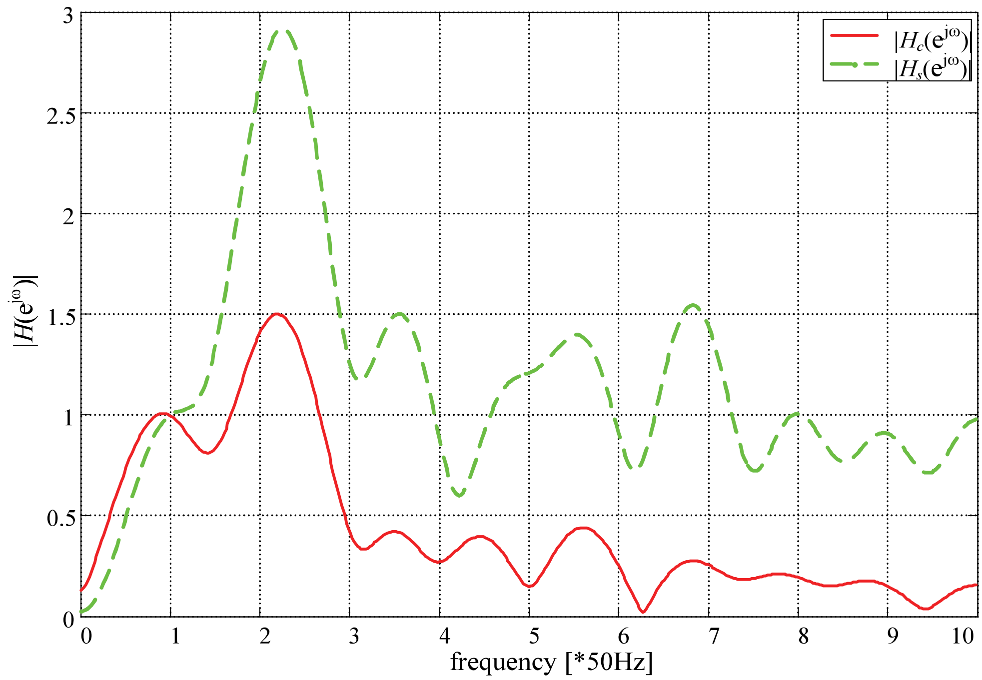

Fig. 4. Frequency response of filters optimized for $M = 7$

Figure 5 presents example estimate of current magnitude for the ground fault in 110 kV line. In addition, estimates realized with the conventional algorithm (1) based on full cycle Fourier filter and based on original and differentiated signal are depicted. As one can see, the response of the optimized estimator shows its immunity to the decaying DC component as well as its rapid reach of the steady state magnitude. The shortening of the estimation transient state duration when compared to response of the conventional algorithm can be assessed to about 12 samples, i.e. 12 ms.

The performance analysis of the three aforementioned measuring algorithms carried out based on the testing signal set allowed for determination of their upper and lower response bounds. Envelopes of response families for unified currents of analyzed estimators are depicted in Fig. 6. It also gives definition of the maximum relative errors used to prepare Fig. 7.

Analyze results for all investigated estimators for M varied from 0 to 20 samples are presented in Fig. 7. The diagram shows that estimators designed for the adopted signal model resulting from fault conditions modelled in 110 kV are characterized by a smaller maximum error starting from $M > 7$ when compared with conventional measuring algorithm. Figure 7 also shows that expanding the length of the transition state above $M = 9$ only slightly reduces the maximum relative errors. The error, as compared with the conventional algorithm, can be for $M = 20$ reduced to about 0.45.

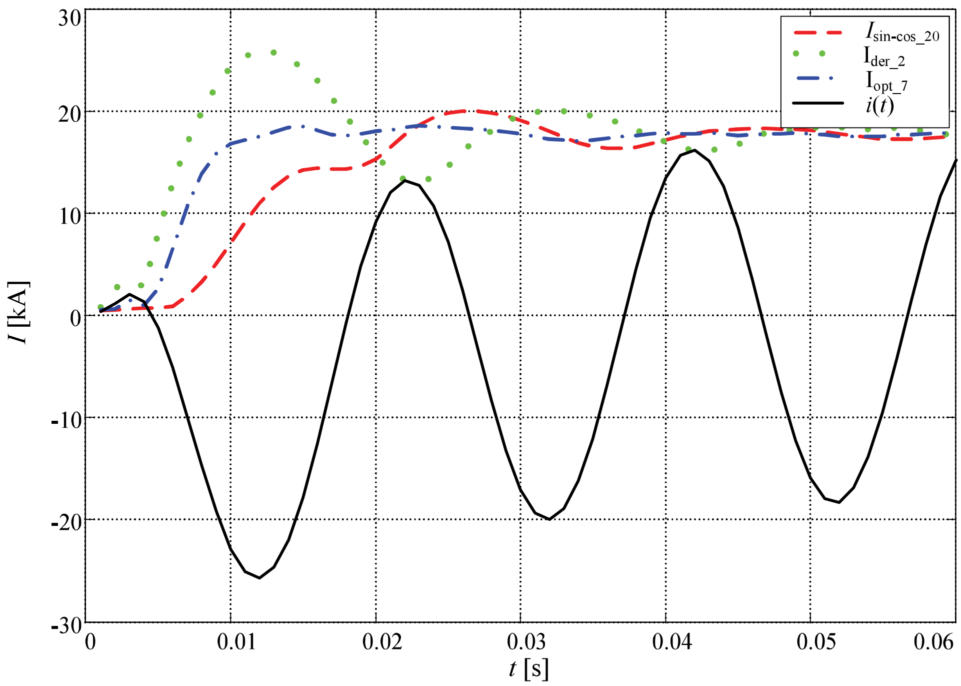

Fig. 5. Current magnitude estimations based on formula (1)
with the use of full cycle Fourier filter,
orthogonalization by signal differentiation, and optimized filters obtained for $M = 7$

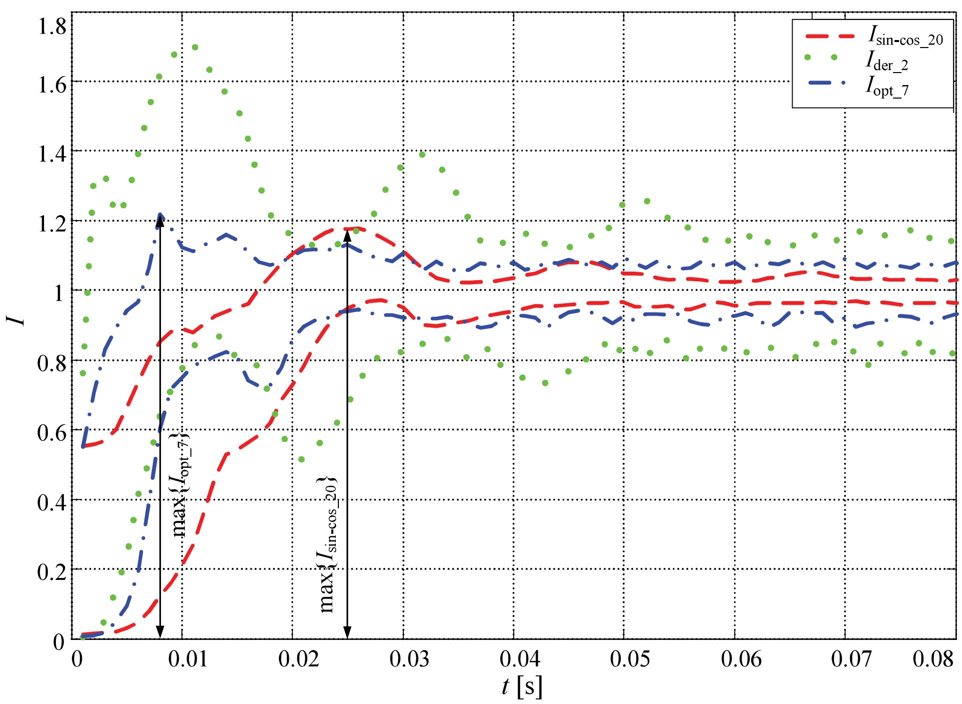

Fig. 6. Transient response limits of the magnitude estimator
based on the full cycle Fourier algorithm,
signal differentiation based algorithm, and the algorithm optimized for $M = 7$

Similar studies were carried out for algorithm optimized for maximum length of the response transient state of 20 samples. The corresponding impulse and frequency responses of the filters are shown in Figs. 8 and 9, respectively. As one can see from comparison of Figs. 5 and 9, resignation from forcing the rapid response allows for better filtering of high frequency components in particular 2nd and 3rd harmonic. The

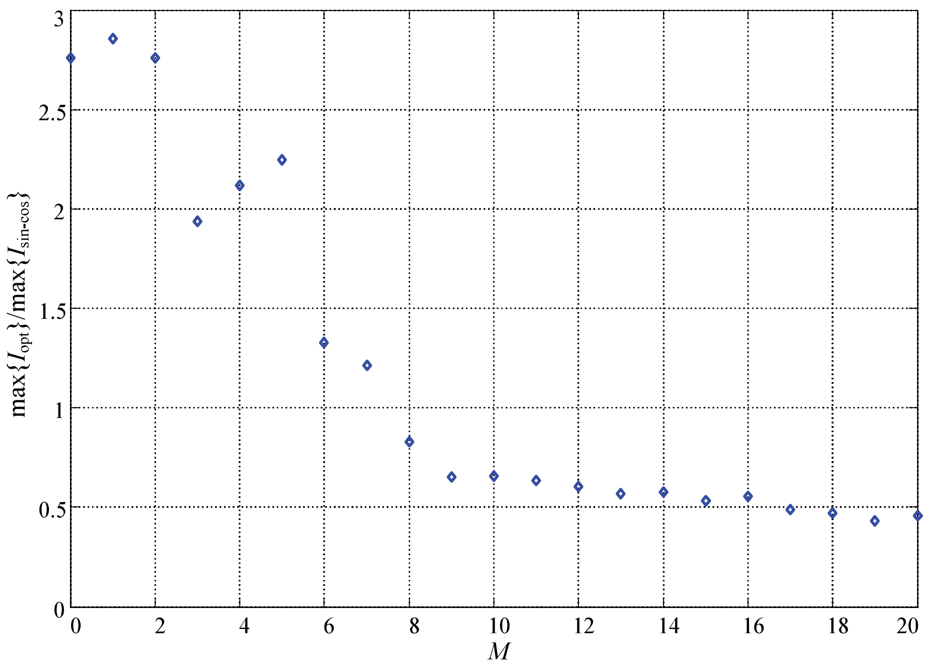

Fig. 7. Maximal relative estimation errors with respect to the error of full cycle Fourier algorithm versus the length of the transient state of M samples

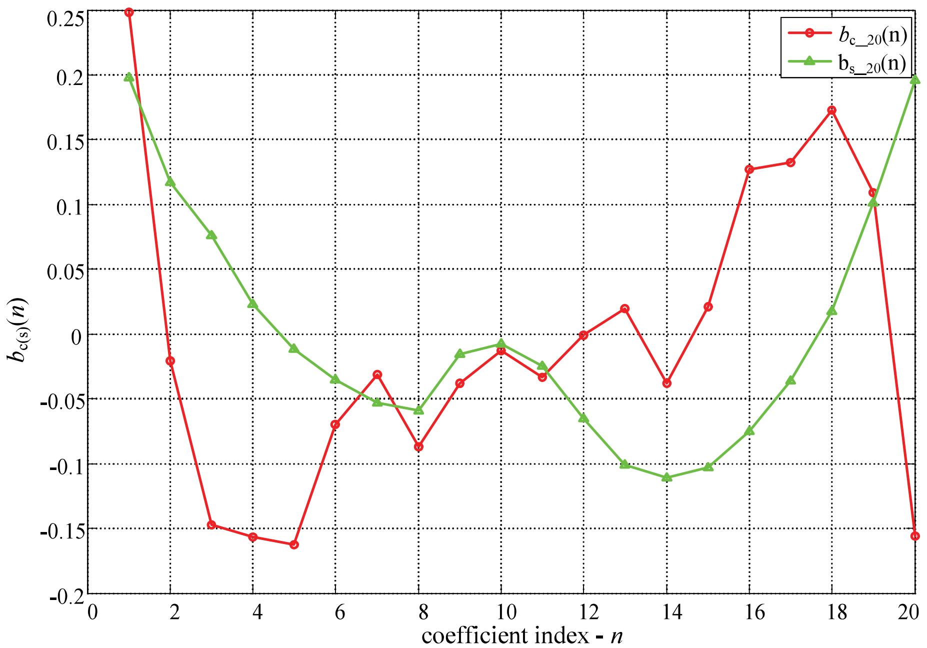

Fig. 8. Unit impulse response of filters optimized for $M = 20$

effect is clearly visible as small errors in the steady state, i.e. narrower envelope in Fig. 10 vs. Fig. 6. However, the ability to eliminate the effect of the decaying DC current component in both temporary and steady state has decisive influence on the quality indicator presented in Fig. 7.

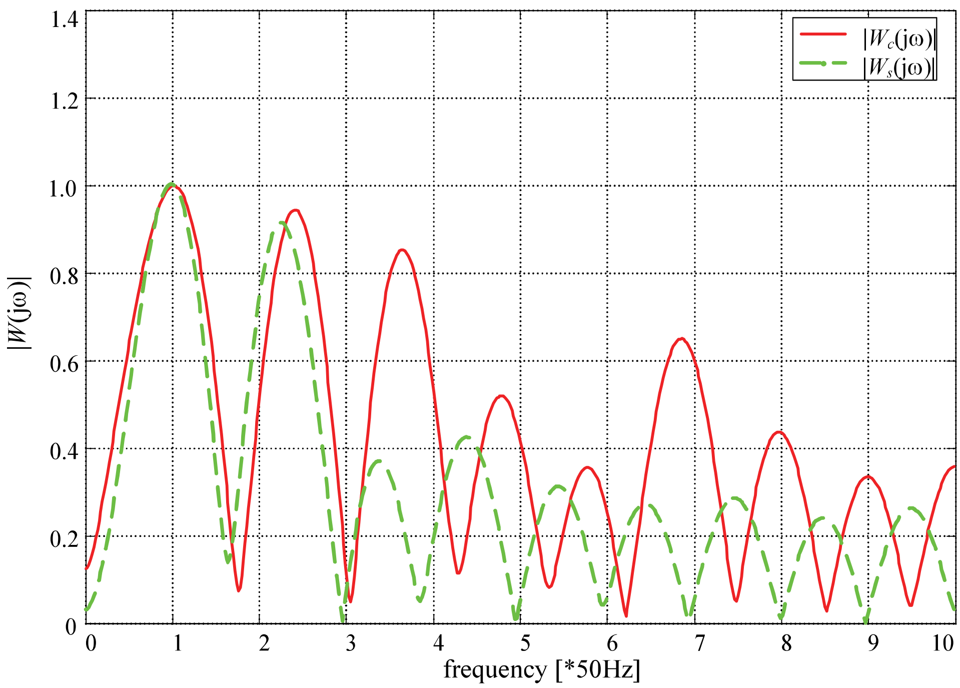

Fig. 9. Frequency response of filters optimized for $M = 20$

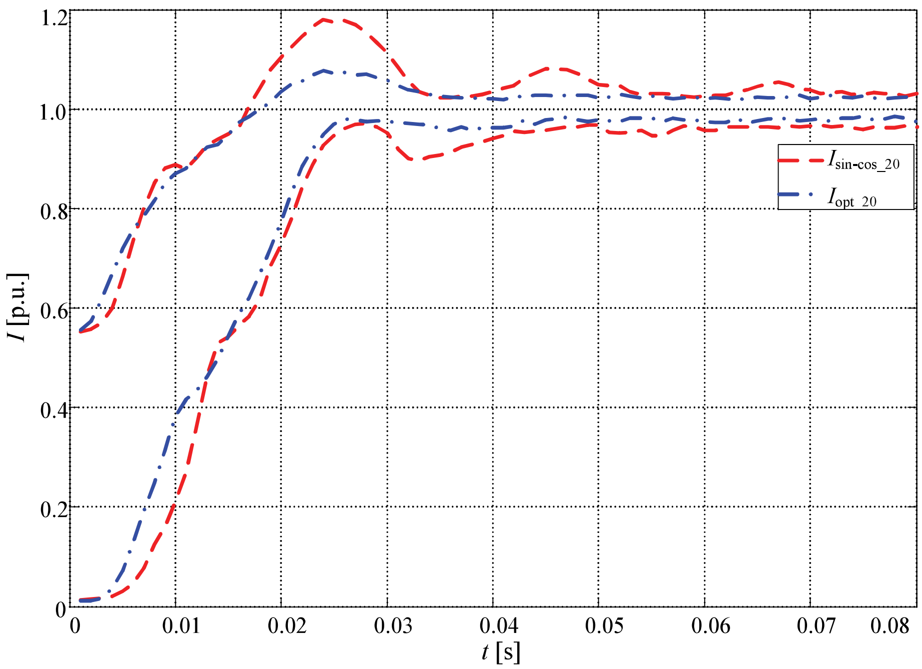

Fig. 10. Transient response limits of the magnitude estimator based on the full cycle Fourier algorithm and the algorithm optimized for $M = 20$

3.2. FREQUENCY DEVIATION IMPACT ON MAGNITUDE ESTIMATION

Since the system frequency undergo continuous fluctuations and is not constant in long time intervals, all estimators of electrical quantities such as magnitude, impedance etc. should be insensitive to frequency deviation in the vicinity of the nominal system frequency. Occurring system frequency variations should not exceed the limits of 49.5 Hz to 50.2 Hz for normal conditions of the power system, however variations in the range 47.0 Hz to 52.0 Hz are accepted.

Figure 11 shows errors of magnitude measurement of purely sinusoidal signal as the function of frequency. As can be seen the error of estimation is zero for full cycle Fourier algorithm, yet only for 50 Hz. The accuracy of this algorithm decreases the deviation pulse, but is still higher when compared with estimates based on the filters designed by means of proposed technique. The accuracy of the conventional method is greater in the frequency range from about 49 Hz to 50.4 Hz. Outside this interval the measuring algorithm based on filters optimized for $M = 7$ is more accurate. One may also notice a relatively higher resistance of the algorithm based on filters optimized for 20 sample transient state. In this case, estimation errors are less than 1% for the frequency deviation of ±3 Hz.

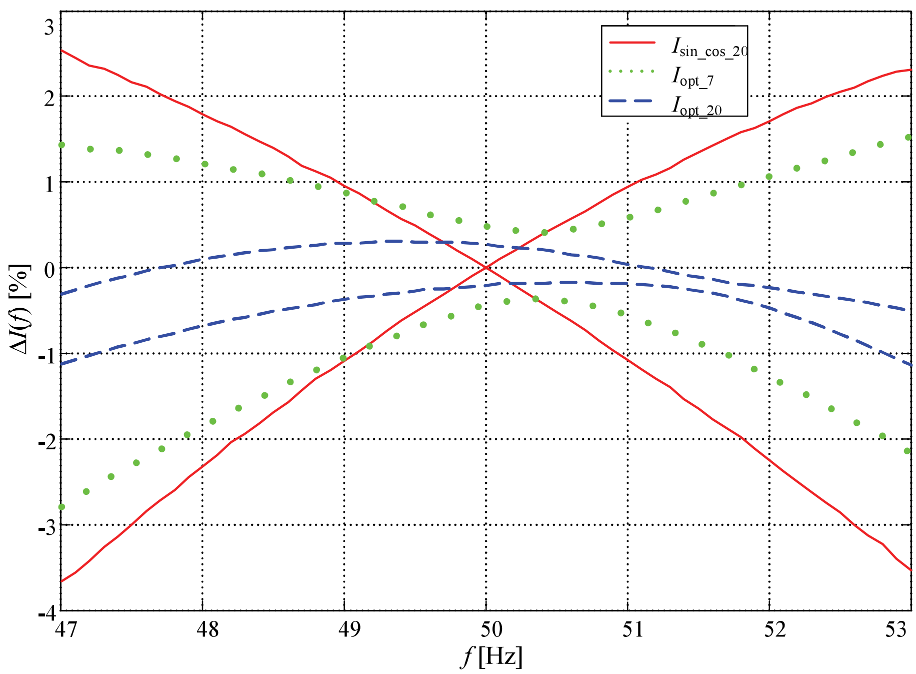

Fig. 11. Magnitude estimation errors vs. deviation of the system frequency

4. CONCLUSION

The paper presents optimization method of magnitude estimators intended for instantaneous overcurrent protections. Assumptions regarding the project were primarily to ensure the prompt estimator response after a fault inception and simultaneous reduction of transient state errors resulting mainly from the decaying DC current component.

By appropriate choice of the signal model, the proposed approach allows for arbitrary forming of magnitude characteristics of the filters. This selection is carried out by selection of the power system model determining transient states of electrical phenomena, the appropriate choice of the fault phenomena model, i.e. arc model, interferences occurring in the normal operation of the power system as well as the impact of instrument transformers on protection quantity wave shapes.

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