**Scientific Papers of
the Department of Electrical Power Engineering of
the Wrocław University of Technology**

PRESENT PROBLEMS OF POWER SYSTEM CONTROL

6

Wrocław 2015

Cover design
Piotr PIERZ

Printed in the camera ready form

Department of Electrical Power Engineering
Wrocław University of Technology
Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland
phone: +48 71 320 35 41
www: http://www.weny.pwr.edu.pl/instytuty,52.dhtml; http://www.psc.pwr.edu.pl
e-mail: wydz.elektryczny@pwr.edu.pl

OFICYNA WYDAWNICZA POLITECHNIKI WROCŁAWSKIEJ
Wybrzeże Wyspiańskiego 27, 50-370 Wrocław
http://www.oficyna.pwr.edu.pl
e-mail: oficwyd@pwr.edu.pl
zamawianie.ksiazek@pwr.edu.pl

ISSN 2084-2201

Print and binding: beta-druk, www.betadruk.pl

composite load model, nonlinear optimization,
nonlinear least squares, genetic algorithms,
sequential quadratic programming

Paweł REGULSKI*

ESTIMATION OF COMPOSITE LOAD MODEL PARAMETERS AS A CONSTRAINED NONLINEAR PROBLEM

This paper presents the results of application of sequential quadratic programming to the estimation of the unknown composite load model parameters. Traditionally applied estimation methods, such as nonlinear least squares or genetic algorithms, suffer from a number of issues. Genetic algorithms exhibit premature convergence and require high computational resources and nonlinear least squares method is very sensitive to the initial guess and can diverge easily. This paper provides a comparison of all three methods based on computer-generated signals serving as field measurements. Accuracy and precision are assessed as well as computational requirements.

1. INTRODUCTION

Loads are one of the most uncertain elements of power systems. They play a key role in power system analysis and inaccurate modeling of loads may result in erroneous assessment of voltage stability [1], [2] as well as other types of studies, such as those on transient stability or load shedding [3], [4]. This becomes unacceptable in the current trend, where environmental considerations push the operating point of power systems closer to their stability limits.

Measurement-based load modeling [4], [5], in which the load characteristics are extracted through a parameter estimation procedure from appropriate field measurements, offers the means for obtaining accurate load models. In such an approach, the aim is to minimize the difference between the output of an assumed load model and the corresponding field measurements. The final reliability and accuracy of the load model relies heavily, among other things, on the selected parameter estimation technique, which is especially true in the case of a composite load (CL) model. The CL

* Wrocław University of Technology, Department of Electrical Power Engineering, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland, email: pawel.regulski@pwr.edu.pl

model is a highly nonlinear model of an induction motor (IM) connected in parallel with a static load. It is used typically to model loads dominated by IMs, which may include not only industrial loads (large machines) but also residential and commercial loads (smaller, single-phase machines). Traditionally, methods such as nonlinear least squares (NLS) [6] or genetic algorithms (GA) [7] have been employed to solve the problem of finding the unknown load model parameter. However, these methods suffer from several flaws. NLS is prone to divergence and, as a deterministic method, is shows strong sensitivity to the initial guess. GA on the other hand, may exhibit premature convergence and it is more time-consuming then NLS. Moreover, both of the methods disregard the fact that the estimation of parameters of the CL model, from its definition, is a constrained problem. This paper addresses this particular issue by applying sequential quadratic programming (SQP) method, which is known to be efficient in solving problems of similar nature [8]. In the area of power systems it has been already successfully applied to solving the hydro unit commitment problem [9] as well as the optimal power flow problem [10].

2. COMPOSITE LOAD MODEL

The CL model is the most complex widely used load model and, according to a recent survey [11], about 30% of utilities around the world use it for dynamic power system studies. It is a voltage dependent model including a 3rd order IM model connected in parallel with a static load model. The IM model adopted in this work can be found in [5] and its full derivation is presented in [2]. The static part of the model is described by an exponential load (EL) model with the following equations:

$$P_S = P_{S0}\left(V/V_0\right)^{\alpha} \tag{1}$$

$$Q_S = Q_{S0}\left(V/V_0\right)^{\beta} \tag{2}$$

where V_0 is the pre-disturbance voltage in pu, P_{S0} and Q_{S0} are the pre-disturbance active and reactive power consumed by the static load, respectively, in W and var. P_S and Q_S are the static load power demands, respectively, in W and var, α and β are the static exponents and V is the actual rms voltage in pu.

The complete vector of unknown model parameters to be estimated is defined as follows:

$$\mathbf{\theta} = [H, R_s, X_s, R_r, X_r, X_m, A, B, K_P, T_0, \alpha, \beta] \tag{3}$$

where H is the inertia constant in s, R_s is the stator resistance in pu, X_s is the stator reactance in pu, R_r is the rotor resistance in pu, X_r is the rotor reactance in pu, $X_{m,}$ is the magnetizing reactance in pu, A and B are the torque coefficients and T_0 is the nominal torque at nominal speed in pu. The parameter K_P is defined as follows:

$$K_P = {P_{M0}}\big/{P_0} \tag{4}$$

where P_{M0} is the initial active power consumed by the IM, in W, and P_0 is the pre-disturbance active power measured at the load bus in W. Table 1 presents the searching space for the parameters defined in (3). It has been selected based on [1] and [2] to cover a wide range of types of motors and characteristics of static loads.

Table 1. Selected ranges of the CL model parameters

	Searching space	
	min	max
H	0.200	2.000
R_s	0.001	0.100
X_s	0.050	0.200
R_r	0.010	0.100
X_r	0.100	0.300
X_m	2.000	4.000
A	0.000	1.000
B	0.000	1.000
K_p	0.200	1.000
T_0	0.200	1.000
α	0.000	4.000
β	0.000	4.000

3. SEQUENTIAL QUADRATIC PROGRAMMING

The SQP approach has been extensively used in 1970s [12]. Its high efficiency and accuracy, when compared to other optimization methods, has been further proved by Schittkowski on a large number of test examples [13]. The SQP provides a framework for solving general nonlinear programming problems of the following form:

$$\min_{\mathbf{x}} f(\mathbf{x}) \tag{5a}$$

subject to

$$\begin{aligned} c_i(\mathbf{x}) = 0, \qquad & i \in \mathcal{E} \\ c_i(\mathbf{x}) \geq 0, \qquad & i \in \mathcal{I} \end{aligned} \tag{5b}$$

where $\mathbf{x} \in \mathbf{R}^n$, f is the objective function, c_i, $i \in \mathcal{E}$ are the equality constraints and c_i, $i \in \mathcal{I}$ are the inequality constraints.

The main concept behind the SQP approach is to model the problem (5) as a sequence of quadratic problems of the following form:

$$\min_{\mathbf{p}} f(\mathbf{x}_k) + \nabla f(\mathbf{x}_k)^T \mathbf{p} + \frac{1}{2}\mathbf{p}^T \nabla^2_{xx} \mathcal{L}_k \mathbf{p} \tag{6a}$$

subject to

$$\begin{aligned} \nabla c_i(x_k)^T \mathbf{p} + c_i(\mathbf{x}) = 0, \qquad & i \in \mathcal{E} \\ \nabla c_i(x_k)^T \mathbf{p} + c_i(\mathbf{x}) \geq 0, \qquad & i \in \mathcal{I} \end{aligned} \tag{6b}$$

where index k is the iteration number, $\mathbf{p}$ is the search direction in the quadratic problem, ∇ is the gradient operator and ∇^2_{xx} is the Hessian operator with respect to $\mathbf{x}$. The Lagrangian function $\mathcal{L}_k$ is described as follows:

$$\mathcal{L}_k = f(\mathbf{x}_k) - \boldsymbol{\lambda}_k^T c(\mathbf{x}_k) \tag{7}$$

where $\boldsymbol{\lambda}_k$ is a vector of Lagrange multipliers.

The quadratic problem (6) is solved using the *active set strategy*, which is an iterative approach. It starts with an initial guess of an active set $\mathcal{A}$, which for any feasible $\mathbf{x}$ is a set of the equality constraint indices $\mathcal{E}$ together with the indices of the inequality constraints for which $c_i(\mathbf{x}) = 0$:

$$\mathcal{A}(\mathbf{x}) = \mathcal{E} \cup \{i \in \mathcal{I} | c_i(\mathbf{x}) = 0\} \tag{8}$$

The *active set strategy* solves this equality constrained problem using the constraints, indices of which are included in the current active set $\mathcal{A}$. In each iteration it performs the following three major tasks:

a) it finds the direction towards the solution and calculates the length of the step,
b) it checks whether the step is violating any constrains, indices of which are not included in $\mathcal{A}$ and adds them to $\mathcal{A}$ if that is the case and
c) it removes the inequality constraint indices from $\mathcal{A}$ if the Lagrange multipliers corresponding to those constraints become negative, which suggests that the objective function can be further minimized by moving away from those constraints.

The above method terminates once the solution to the quadratic sub-problem is equal to 0 (the calculated direction increment at the current iteration is equal to 0) [8].

The final solution of the quadratic problem (6) is used to update the $\mathbf{x}_k$:

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha \mathbf{p} \tag{9}$$

where α is the step-length coefficient obtained using a line search approach to minimize a merit function, which ensures a sufficient decrease in the objective function [8]. After that, the iteration number k is incremented and a new quadratic problem is formulated. The procedure repeats until $\mathbf{x}_k$ meets the Karush–Kuhn–Tucker (KKT) optimality conditions [8].

The above paragraphs gave an introduction to what the sequential quadratic programing is. However, to implement it for solving the problem of estimation of unknown load model parameters, the suitable objective function of the problem must be defined. It must be followed with appropriate constraints.

For the CL model, the objective function for a given n samples of input data, i.e. measurements, can be defined as follows:

$$\min_{\boldsymbol{\theta}} \frac{1}{n} \sum_{k=1}^{n} [(P_{mk} - P_{CLMk}(\boldsymbol{\theta}))^2 + (Q_{mk} - Q_{CLMk}(\boldsymbol{\theta}))^2] \tag{10}$$

subject to the bound constraints defined in Table 1 and the inequality $A + B \leq 1$ imposed on the torque coefficients [5]. P_{mk} and Q_{mk} are the k-th sample of measured active and reactive powers, respectively, in W and var and P_{CLMk} and Q_{CLMk} are the k-th sample of estimated active and reactive powers, respectively, in W and var.

4. RESULTS

SQP has been tested and compared against the traditional methods (NLS and GA) in a series of computer simulations. Firstly, 6 test voltage signals have been generated to excite the CL model. These include step changes and ramps of different magnitudes and are depicted in Figure 1.

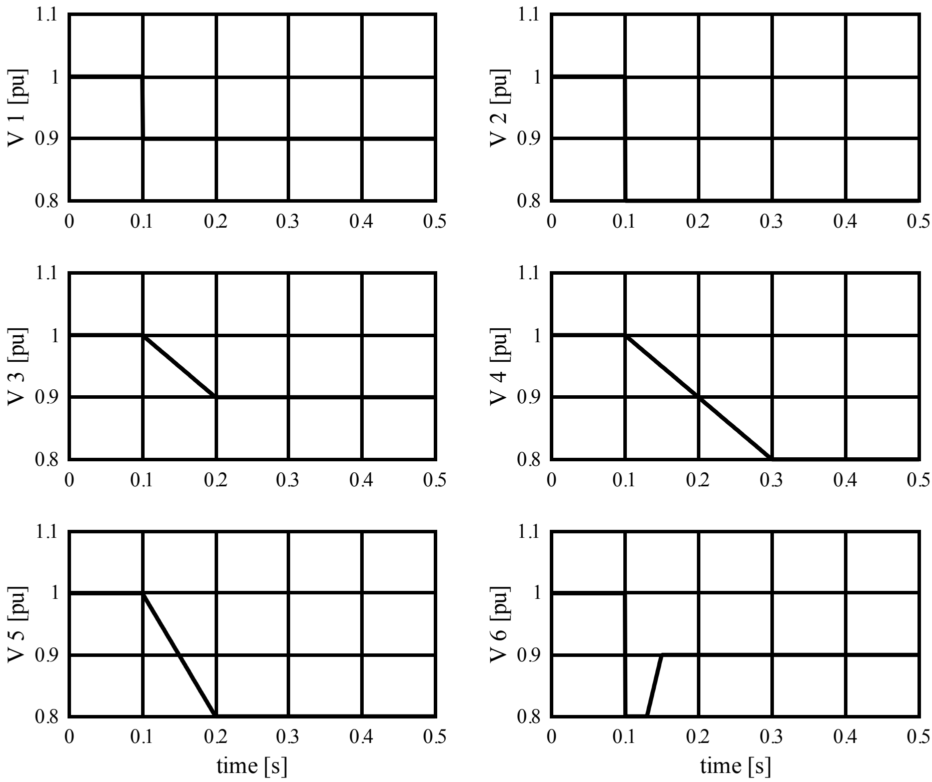

Fig. 1. Voltage signals used to excite the CL model

Based on the 6 generated voltage signals, active and reactive power responses of the CL model were simulated for a given set of parameters. These responses are depicted in Figure 2. Such a set of computer-generated signals was assumed as field measurements for the purpose of the estimation process.

Initial conditions for each method were obtained from the same set of 100 parameter vectors randomly generated from the assumed searching space (Table 1). In the case of SQP and NLS, each vector was used as a starting point of the procedure, for a total of 100 runs for each method. On the other hand, GA uses the whole set of vectors as an initial population. However, due to the nature of the method, which is driven to an extent by random processes, the estimation procedure has been repeated 100 times to assess its average performance. In this way, each method returned 100 solutions, which allowed for a reasonable comparison.

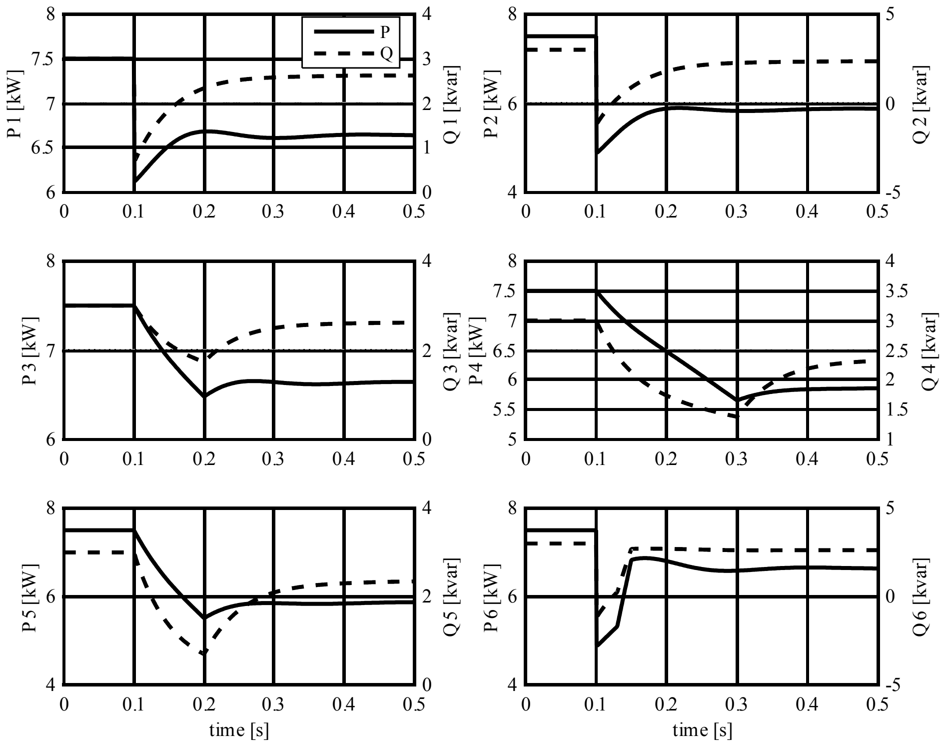

Fig. 2. Active and reactive power responses of the assumed CL model

In the estimation process, 2 out of 6 data sets have been used. That includes measurement 2 (voltage step change) and measurement 4 (voltage ramp). This allows covering a wider range of the model's responses and improves the generality of the identified parameters. It is important to notice that using insufficient amount of data at the training stage may result in loss of generality (overtraining), which manifests itself in a very good fit at the training stage and a poor fit at the validation stage. In this case, the estimation result has been validated against all 6 data sets (cases) and the results have been presented in Tables 2 and 3. Firstly, it can be observed that SQP achieves the smallest average relative errors in 100 trials and NLS achieves the highest (Table 2). Secondly, standard deviation of the relative errors presented in Table 2 shows that SQP is also the most consistent method, which makes it more accurate and precise than the other two approaches. The results also confirm that NLS is very sensitive to the initial guess and can easily diverge. Figure 3 depicts comparison of all 3 methods in their best trial. It can be concluded that NLS can achieve accuracy similar to that of SQP, but only if the initial guess is appropriately selected.

Table 2. Average relative errors obtained for each case (based on 100 trials)

	GA		NLS		SQP	
	Perr [%]	Qerr [%]	Perr [%]	Qerr [%]	Perr [%]	Qerr [%]
Case 1	0,163	0,645	2,042	7,803	0,009	0,081
Case 2	0,240	1,234	2,813	18,919	0,008	0,057
Case 3	0,139	0,489	1,810	7,012	0,006	0,065
Case 4	0,136	0,628	1,988	12,018	0,006	0,038
Case 5	0,185	0,949	2,370	15,905	0,007	0,041
Case 6	0,195	0,784	2,448	9,179	0,011	0,092

Table 3. Standard deviation of the relative errors obtained for each case (based on 100 trials)

	GA		NLS		SQP	
	Std. Perr	Std. Qerr	Std. Perr	Std. Qerr	Std. Perr	Std. Qerr
Case 1	0,237	0,420	3,064	14,079	0,010	0,047
Case 2	0,363	1,245	3,744	36,769	0,014	0,086
Case 3	0,170	0,356	3,011	13,780	0,009	0,038
Case 4	0,155	0,824	2,815	23,531	0,009	0,066
Case 5	0,254	1,096	3,212	31,190	0,010	0,079
Case 6	0,320	0,486	3,293	16,596	0,014	0,053

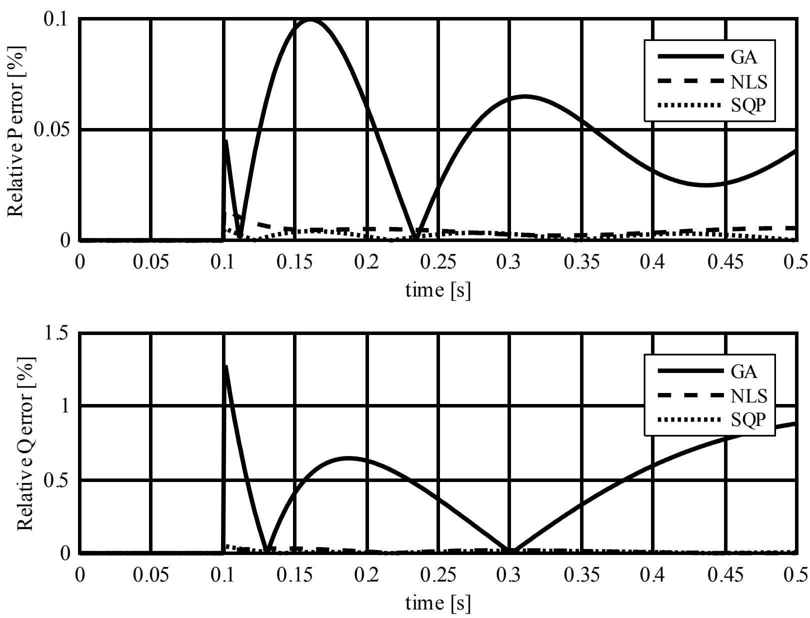

Fig. 3. Comparison of best estimations achieved by each method (validation using case 2)

The selection of the training set in this particular case turned out to be successful. There are no obvious discrepancies between average relative errors obtained for training cases and validation cases (Table 2). Other combinations of cases forming the training set have been tested with very similar results. It has been also observed that voltage step changes reveal more dynamic properties than voltage ramps and it advisable always to include a voltage step change in the training set for more reliable results.

In terms of computational performance, the average execution times for each method are presented in Table 4. In this particular test, SQP is over two times slower than NLS, but still much faster than GA. It should also be noted that the implementation of GA, in this case, takes advantage of parallel computing and it utilizes 4 available cores of the CPU. On a single-core CPU the average execution time of GA would be approximately 4 times higher.

Table 4. Average execution times in seconds
(based on 100 trials)

GA	NLS	SQP
184	9	23

5. CONCLUSIONS

This paper proposed the application of SQP to estimate the unknown parameters of the CL model from field measurements. This method takes into account constrains, which was impossible with the use of traditional methods such as NLS or GA. SQP achieves highest accuracy and precision without the loss of the generality of the assumed load model. Its execution time, although higher than that of NLS, still allows for online load monitoring with the assumption that appropriate voltage disturbances do not occur too often.

The execution time of SQP, when compared to NLS, reflects its more complex implementation, which might be recognized as a disadvantage. However, the benefits of this approach outweigh this flaw significantly.

The preliminary tests presented in this paper have provided very promising results. In the next step, SQP should be examined using either laboratory or field measurements to ensure that the benefits of using SQP can also be achieved in practice.

REFERENCES

[1] VAN CUTSEM T., VOURNAS C., *Voltage Stability of Electrical Power System*, Kluwer Academic Publishers, 1998.

[2] KUNDUR P., *Power System Stability and Control*, McGraw-Hill, 1994.

[3] RIFAAT R.M., *On Composite Load Modeling for Voltage Stability and Under Voltage Load Shedding*, Power Engineering Society General Meeting, 2004, IEEE, June 2004, Vol. 2, pp. 1603–1610.

[4] JU P., WU F., SHAO Z.-Y., ZHANG X.-P., FU H.-J., ZHANG P.-F., HE N.-Q., HAN J.-D., *Composite Load Models Based on Field Measurements and Their Application in Dynamic Analysis*, Generation, Transmission and Distribution, IET, Sept. 2007, Vol. 1, No. 5, pp. 724–730.

[5] MA J., DONG Z.-Y., HE R.-M., HILL D.J., *Measurement-based Load Modeling using Genetic Algorithms*, IEEE Congress on Evolutionary Computation 2007. CEC 2007, Sept. 2007, 25–28, pp. 2909– 2916.

[6] WANG J.-C., CHIANG H.-D., CHANG C.-L., LIU A.-H., HUANG C.-H., HUANG C.-Y., *Development of a frequency-dependent composite load model using the measurement approach*, IEEE Transactions on Power Systems, Aug. 1994, Vol. 9, No. 3, pp. 1546–1556.

[7] WEN J.Y., JIANG L., WU Q.H., CHENG S.J., *Power System Load Modeling by Learning Based on System Measurements*, IEEE Transactions on Power Delivery, April 2003, Vol. 18, No. 2, pp. 364–371.

[8] NOCEDAL J., WRIGHT S. J., *Numerical Optimization*, 2nd Ed., Springer, 2006.

[9] FINARDI E.C., DA SILVA E.L., *Solving the hydro unit commitment problem via dual decomposition and sequential quadratic programming*, IEEE Transactions on Power Systems, May 2006, Vol. 21, No. 2, pp. 835–844.

[10] SIVASUBRAMANI S., SWARUP K.S., *Sequential quadratic programming based differential evolution algorithm for optimal power flow problem*, Generation, Transmission and Distribution, IET, Nov. 2011, Vol. 5, No. 11, pp. 1149, 1154.

[11] MILANOVIC J.V., YAMASHITA K., MARTINEZ VILLANUEVA S., DJOKIC S.Z., KORUNOVIC L.M., *International Industry Practice on Power System Load Modeling*, IEEE Transactions on Power Systems, Aug. 2013, Vol. 28, No. 3, pp. 3038–3046.

[12] BIGGS M.C., *Constrained Minimization Using Recursive Quadratic Programming*, Towards Global Optimization, North-Holland 1975, pp. 341–349.

[13] HOCK W., SCHITTKOWSKI K., *Test Examples for Nonlinear Programming Codes*, Lecture Notes in Economics and Mathematical Systems, Vol. 187, Springer, 1981.